WOODS HOLE OCEANOGRAPHIC INSTITUTION

Reference No. 51-99

An Experimental Study of

Salt Wedges

WOODS HOLE, MASSACHUSETTS

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Woods Hole, Massachusetts

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An Experimental Study of Salt Wedges

by

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Director

Introduction

The objective of the estuary studies being conducted at this Institution is to attempt an understanding of tidal flushing. Two theories have been developed, that of Ketchum (1) employing an extension of the tidal prism technique, and that of Arons and Stommel (2) who treat the flushing problem through the diffusion theory. An experimental program has been established to determine the validity of these theories so that model laws may be established for tidal flushing.

At the beginning of this program, the phenomenon of the salt water wedge was observed in the experimental apparatus over a wide range of flow conditions. To produce salinity distributions other than this, it was necessary to stimulate diffusion of the salt water with turbulence created by added roughness.

In view of the apparent importance of the salt wedge, it was decided that an investigation of the factors that influence its shape and length would be of particular value to the future work.

An empirical equation has been developed which describes the salt water wedges formed in the laboratory. This equation, however, does not describe well the salt wedge as found in the

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(1) B. H. Ketchum, "The Exchanges of Fresh and Salt Waters in Tidal Estuaries", Journal of Marine Research, Vol. X, No. 1, June 30, 1951.

(2) A. B. Arons and H. Stommel, "A Mixing Length Theory of Tidal Flushing", Transactions, American Geophysical Union, Vol. 32, No. 3, June 1951.

Mississippi River. It is of particular interest that the equation appears to describe the mean horizontal distribution of salinity in estuaries where the fresh and salt water are well mixed and not stratified as in the wedge.

The Salt Water Wedge

Where rivers discharge directly into the ocean or into a bay open to the ocean, it is a well known fact that the salt water from the ocean intrudes upstream into the river channel and mixes with the river water. There is established a certain region of brackish water including the river and bay, in which the mean density of the water over a vertical cross section will vary from that of the river fresh water to the salt water in the ocean.

Various factors which influence the mixing in an estuary are:

(a) the river or fresh water velocity

(b) the mean total depth of water

(c) the bottom and side roughness

(d) irregularities in the over-all shape of the estuary

(e) the difference in density between fresh and salt water, and

(f) the tidal currents.

The maximum distance the salt water intrudes upstream will also depend on these factors. Salt water, because of its greater density, will always tend to underlie the fresh water. When the turbulence generated by the above factors becomes sufficiently great the fresh and salt waters mix and a density or salinity gradient will exist in the horizontal and vertical directions. When the turbulence is slight there is essentially an undiluted body of salt water underlying the outward flowing fresh water.

This mass of salt water, because of its shape, has been called the salt water wedge.

In the laboratory it was observed that the influence of the tides is to cause the salt wedge to change its shape continuously as well as to promote mixing between the salt water in the wedge and the fresh water. The tides are necessary for the conditions to be similar to those in a natural estuary. However, to obtain data on the wedge profile under these conditions was found to be extremely difficult and tedious. The stationary salt wedge, obtained by eliminating the tides, was therefore selected for study. In this case the mixing was reduced and the time variable removed.

Various observations of the salt water wedge were made in the laboratory. Some of these observations are as follows:

(a) Three parameters could be varied to effect different wedges in the flume - the total depth of water, the discharge of fresh water or mixed fresh and sea water, and the density differences. For a given set of these conditions the wedge would establish itself in an equilibrium position and remain stationary. See Fig. 1.

(b) Internal waves were observed at the interface or boundary between the wedge and the upper fluid. Generally the waves occurred over the full length of the wedge. For the A series of data, in which the upper fluid was fresh water and the density difference a maximum. the internal waves were breaking all along the wedge. As the density difference was decreased, the B series of data, the breaking waves occurred over only the deeper portions of the wedge. As a result of the breaking waves there was a small transport of salt water across the interface.

Fig. 2 shows the internal waves forming on the upstream end of a salt wedge. The salt water has been dyed with fluorescine to distinguish the wedge from the fresh water. The leading edge of the wedge is at the extreme right and the free surface is just below the top of the picture.

(c) Fig. 3 illustrates how the velocities of the water varied over the total depth of flow. In the upper layer the velocity of the fresh water is essentially uniform with depth. Within the wedge the salt water near the interface flows in the same direction as the fresh water above. Over the remaining depth of the wedge the salt water reverses its direction of motion and flows upstream. The maximum velocity in this reverse current was estimated as about an order of magnitude smaller than that of the fresh water. The change in velocity with depth is greatest near the interface, indicating a pronounced shearing stress in this region.

(d) The fluid motions within the wedge appear to be laminar. Near the interface turbulence is produced by the breaking internal waves. In the upper layer the fluid motions are in the transition range between laminar and complete turbulent flow.

(e) The slope of the free surface was undetectable by point gage measurement (minimum reading -0.2 mm).

(f) The slope of the interface at the leading edge of the wedge appeared to be infinite. Progressing towards the deep end of the wedge the curvature of the interface was concave downwards for approximately three-quarters the length of the wedge. The curvature then reversed and was concave upwards. These curvatures were generally very small and were often obscured by the experimental error when taking the wedge profile data.

SALT WATER WEDGE

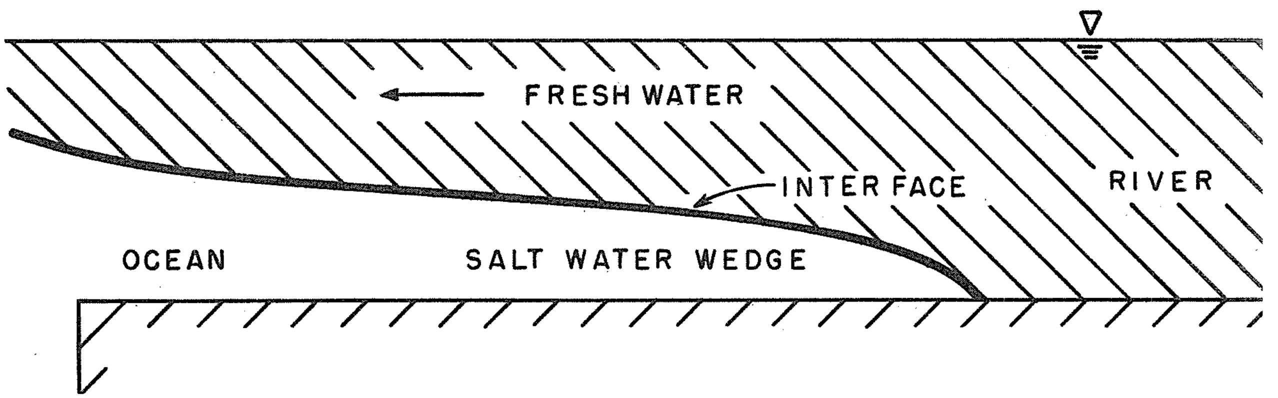


FIG. I

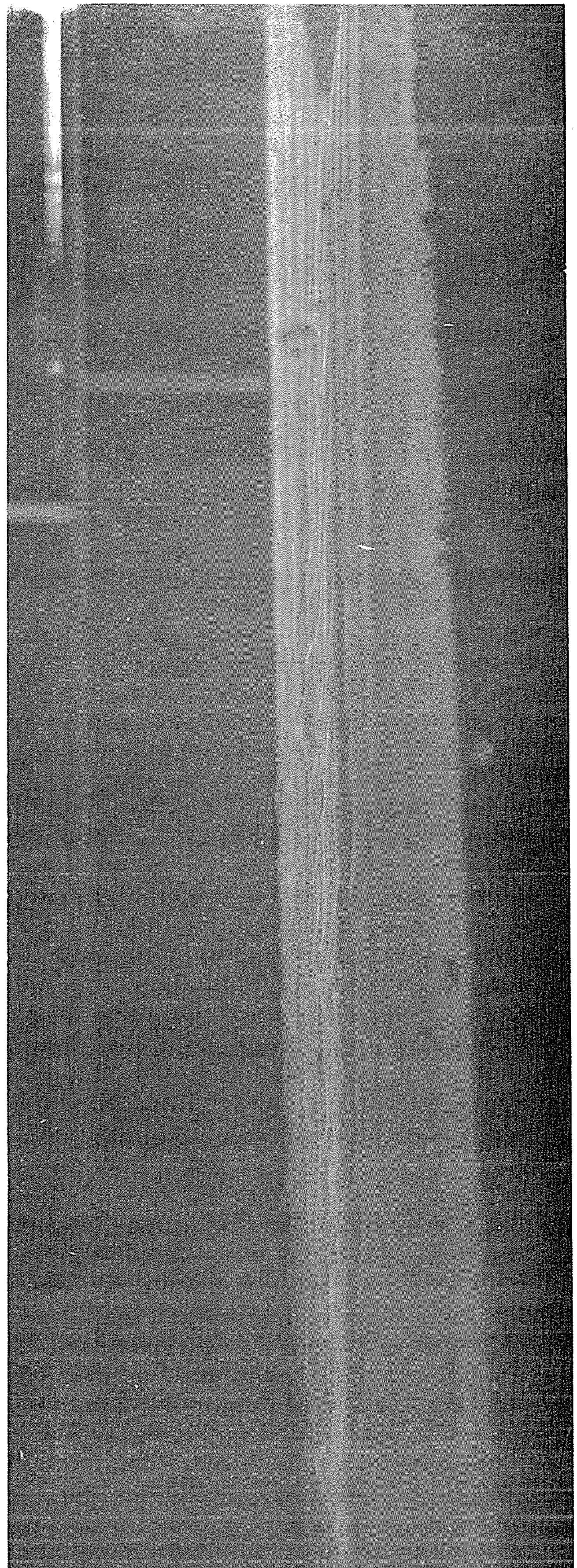


FIG.2

VELOCITY PROFILES OVER VERTICAL CROSS SECTION

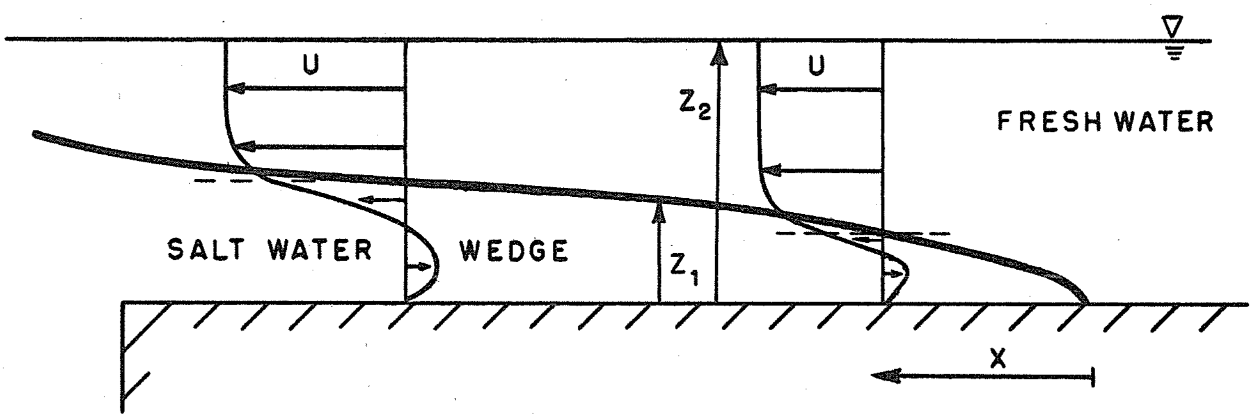


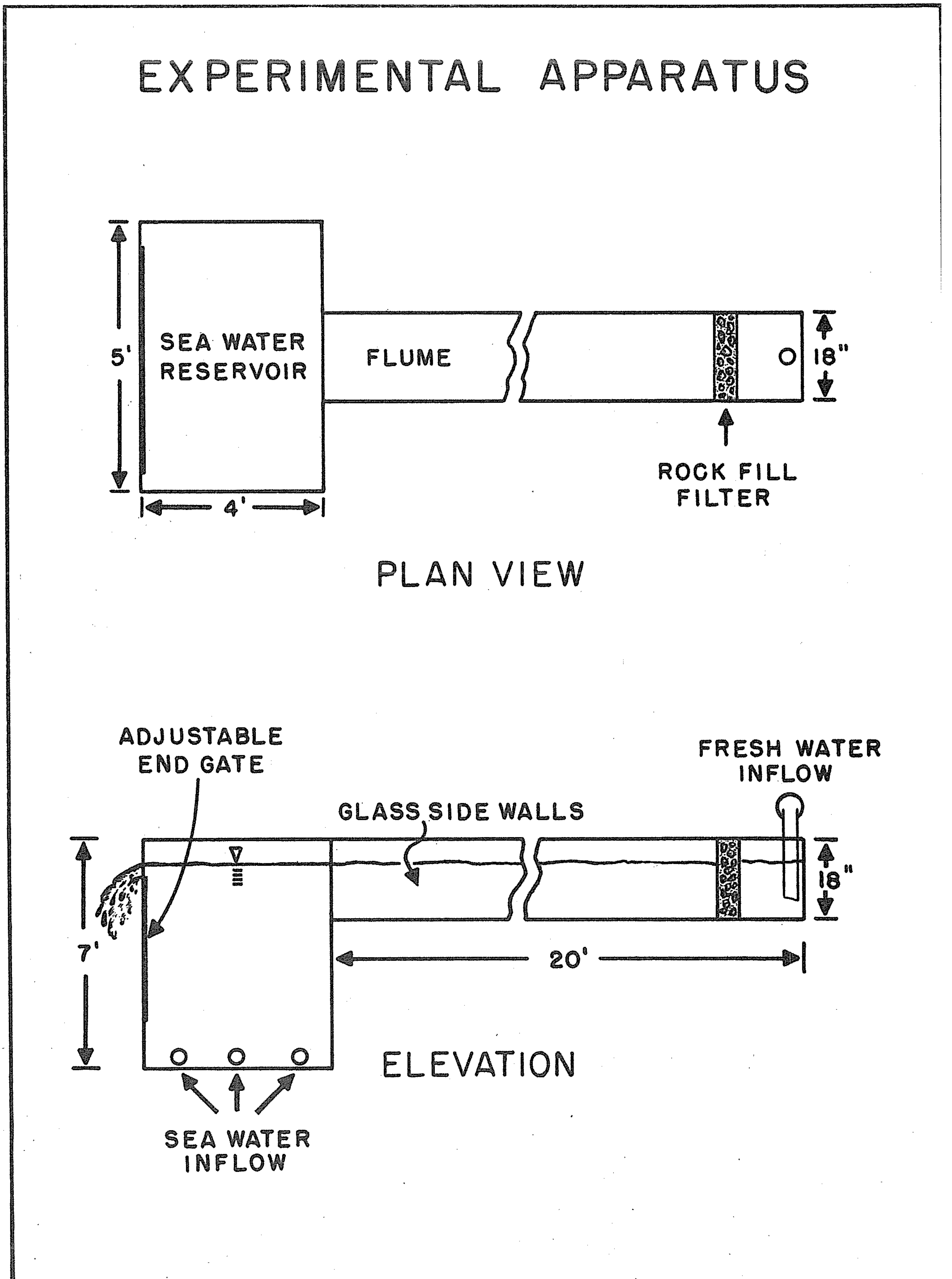
FIG. 3

Experimental Apparatus and Technique

Fig. 4 shows in outline form the experimental apparatus used in the salt wedge studies. The flume has one-quarter inch plate glass side walls and a level bottom made of one and one-half inch redwood. The reservoir and flume are rigidly connected together through a 6 inch long elliptically shaped transition. The total depth of water in the reservoir and flume is controlled by the weir on the overflow side of the reservoir. The weir may be freely adjusted to any height. For tidal studies the weir is raised and lowered sinusoidally by means of the mechanical apparatus mounted over the reservoir, Fig. 5. This consists of a motor driven hydraulic transmission, speed reductor, and a modified Scotch yoke. The flume, in more detail, may be seen in Fig. 6.

The concrete reservoir, representing the ocean source in the apparatus, is constantly supplied with sea water pumped from the Institution dock at a rate up to 180 gpm. The salinity of the sea water remains at a satisfactorily constant 32 $^{o}/oo$ and its density is determined from its temperature and salinity. Fresh water is metered with a Rotameter before discharging into the upstream end of the flume. In order to vary the density difference of the two waters in the flume, sea water may be drawn from the concrete reservoir source, separately metered and then thoroughly mixed with the fresh water before discharge into the flume. Fig. 7 shows the two Rotameters used for this purpose. The specific gravity of the fresh and salt water mixtures was taken by hydrometer and these values were used as density.

The wedge profiles were measured with point gages mounted on level tracks running the length of the flume. The level of the interface was taken in the troughs of the internal waves. When the interface became quite rough, due to the internal waves breaking, see Fig. 8, this level had to be estimated. Under these conditions the error in determining the depth of the wedge may have been large. The average error is estimated at about 5%-10%.



EXPERIMENTAL APPARATUS
5'
SEA WATER
RESERVOIR
FLUME
18"
4'
ROCK FILL
FILTER
PLAN VIEW
ADJUSTABLE
END GATE
FRESH WATER
INFLOW
GLASS SIDE WALLS
7'
18"
20'
ELEVATION
SEA WATER
INFLOW

FIG. 4

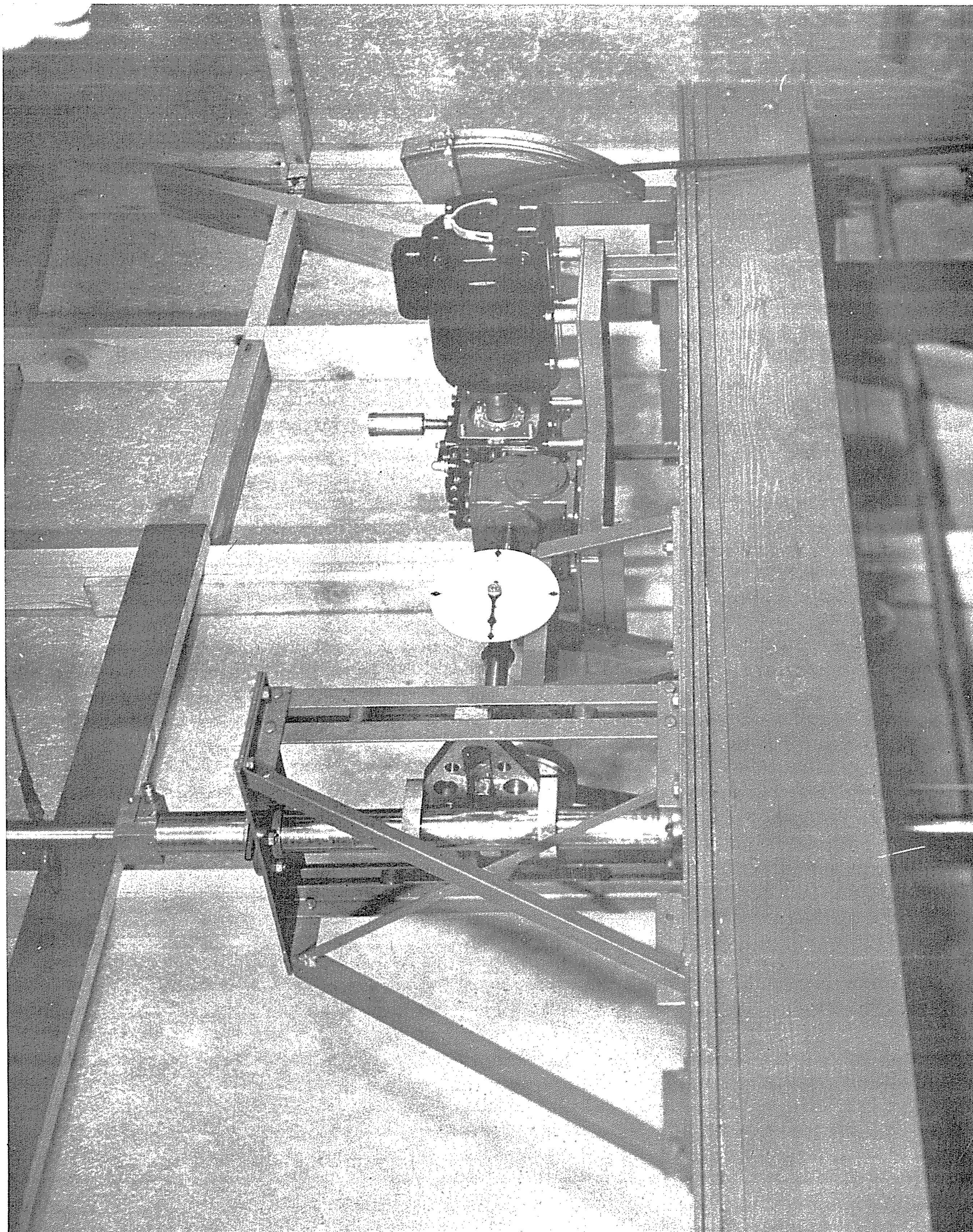


FIG. 5

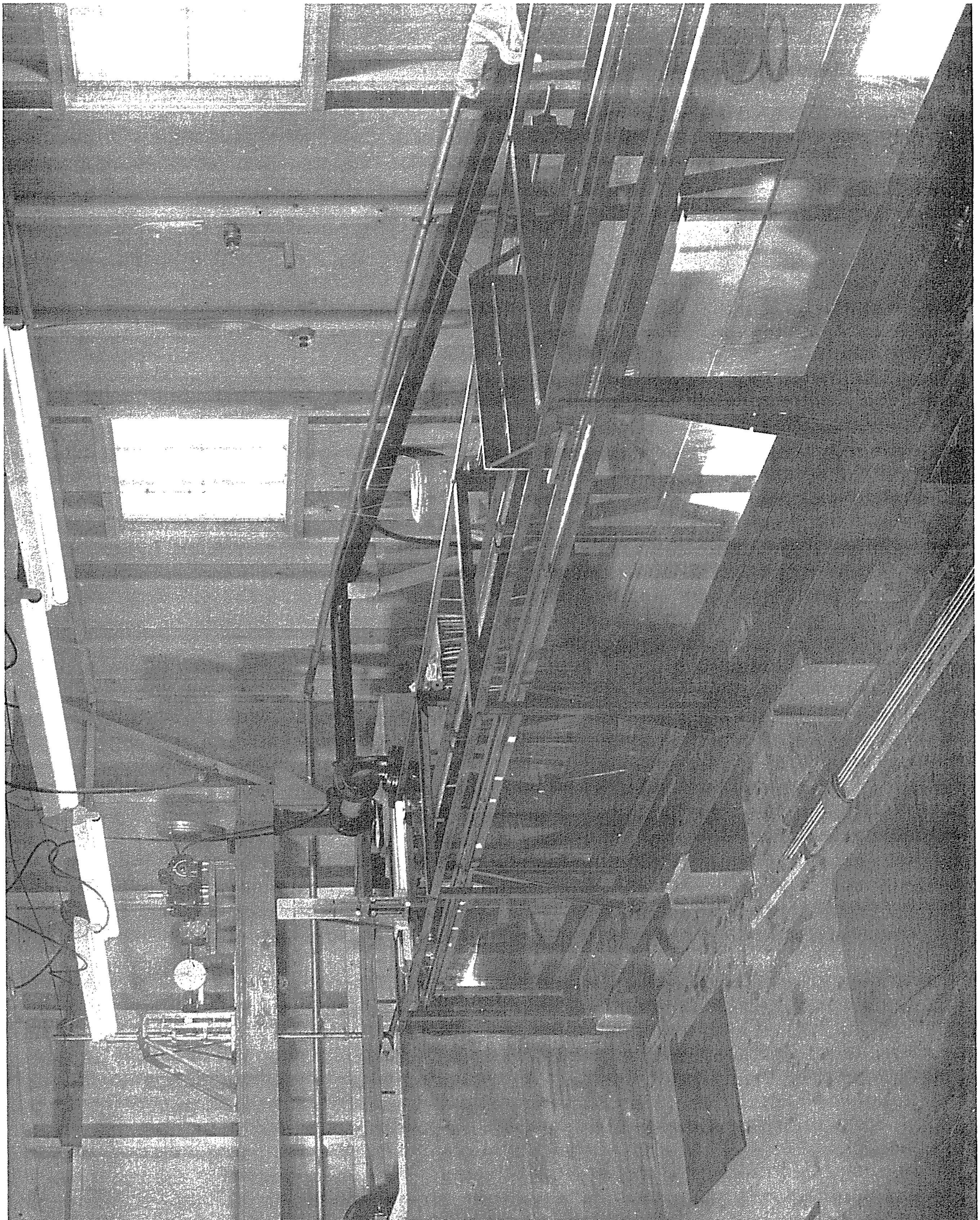


FIG.6

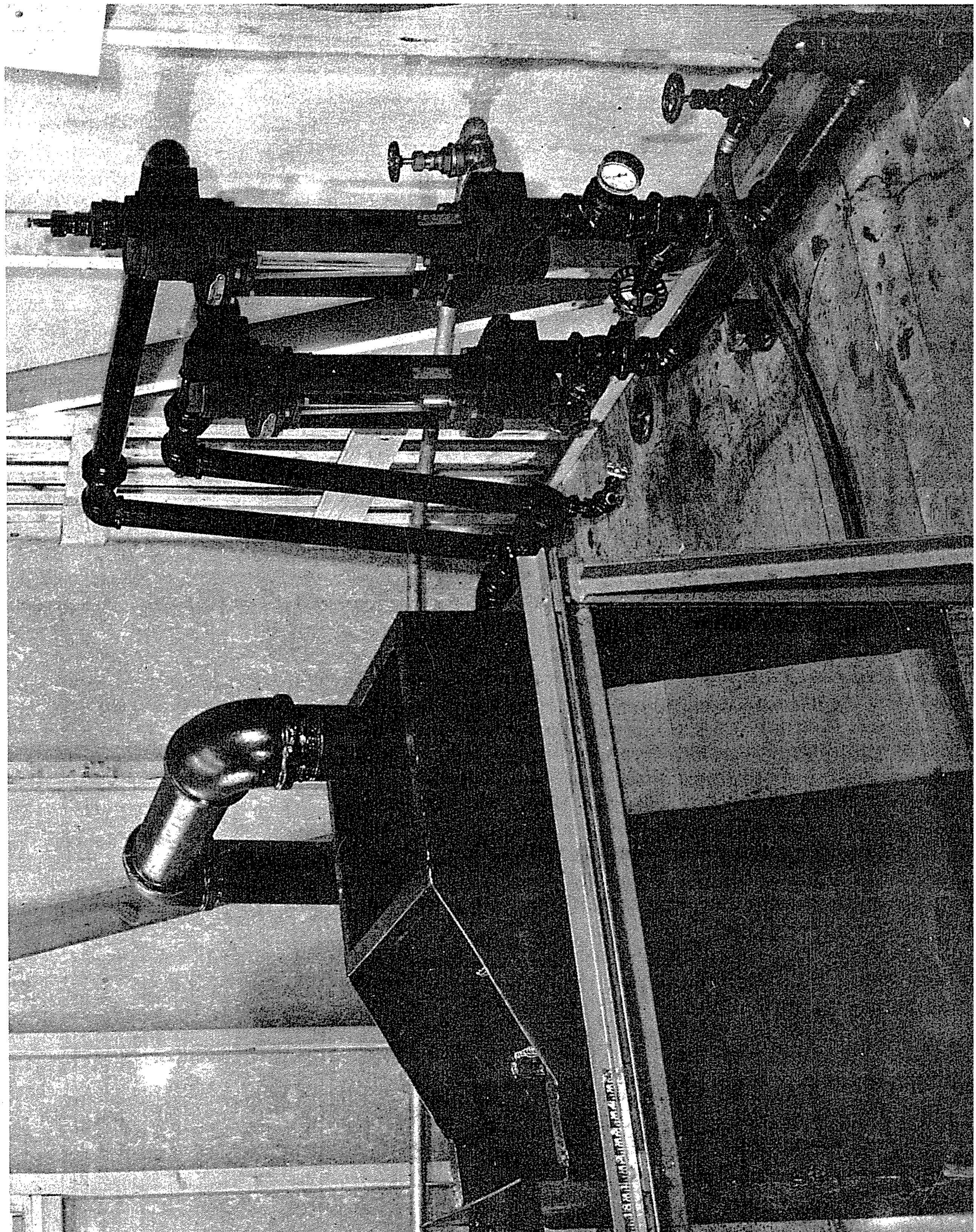


FIG. 7

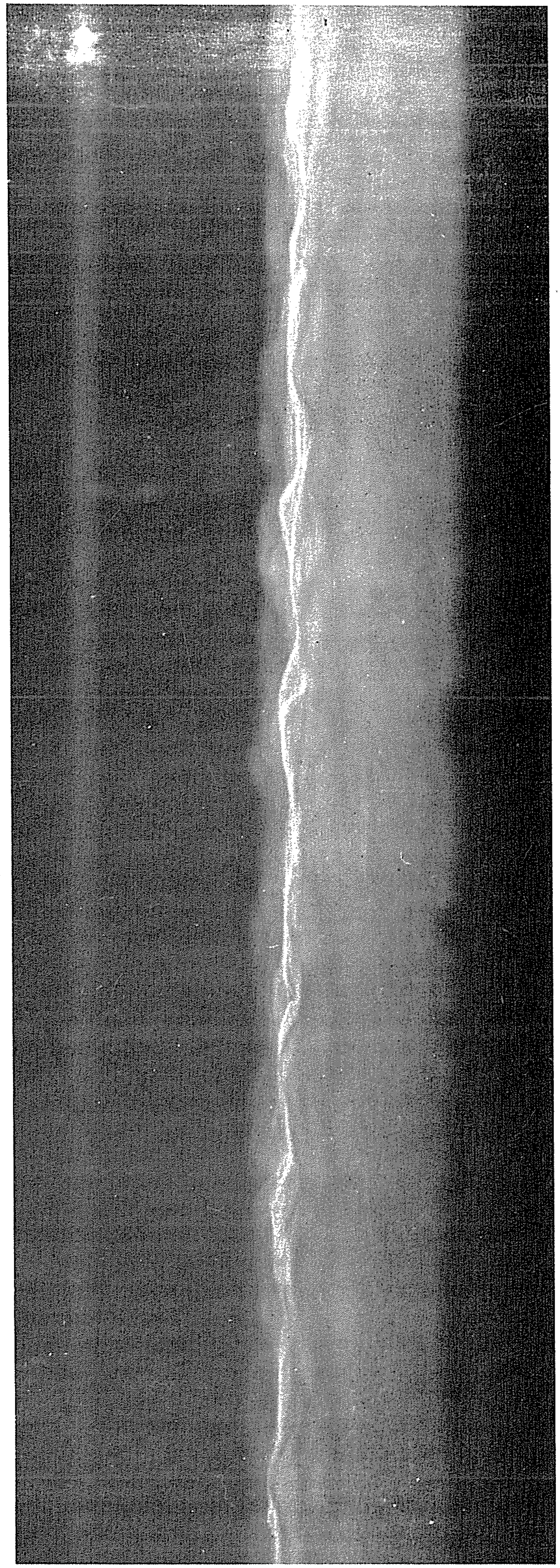


FIG. 8

Discussion of Experimental Data

A total of 45 different salt wedges was established in the flume. The data on the profile and flume conditions for each wedge is given in the Appendix. In the A series of data the wedges were established maintaining a constant density difference, whereas in the B series this parameter was varied. The nomenclature used is summarized below and reference may be made to Fig. 3.

X Horizontal distance measured from leading edge of wedge.

Z_1 Depth of salt wedge measured from the bottom to the interface.

Z_2 Total depth of water measured from the bottom to the free surface.

U Mean velocity of the water in the upper layer at any cross section.

U_0 Mean velocity of the water in the upper layer at $X = 0$.

q $= U(Z_2 - Z_1)$, the discharge per unit width of water in upper layer, a constant for each wedge.

ρ Density of water in upper layer.

ρ_1 Density of the salt water in the wedge.

$\Delta\rho$ $= \rho_1 - \rho$

g The acceleration of gravity.

A plot of the wedge profiles, using the dimensionless coordinates Z_1/Z_2 and X/Z_2 , indicated that each wedge followed approximately a parabolic law. It was further noted that the various profiles were systematically spaced along the X/Z_2 axis, the parameter appearing to be the Froude number corrected for the density difference. When the coordinates were changed to

$$\frac{Z_1}{Z_2} \quad \text{and} \quad \frac{X}{Z_2}\,\frac{U_0^2\,\rho}{g Z_2 \Delta\rho}$$

the data converged so that a single curve approximated all the profiles, Fig. 9 and 10. The relationship indicated by these two plots is

$$\frac{Z_1}{Z_2} = k\left(\frac{X}{Z_2}\,\frac{U_0^2\,\rho}{g Z_2 \Delta\rho}\right)^m \tag{1a}$$

or

$$\frac{Z_1}{Z_2} = k\left(\frac{X}{Z_2^4}\,\frac{q^2\,\rho}{g \Delta\rho}\right)^m \tag{1b}$$

Attempts to incorporate in this relation some form of a Reynolds number, to account for the effects of viscosity, have so far proved unsuccessful.

The spread of the experimental points may be explained, in part, with the aid of Fig. 10. The solid line indicates the wedge profile as given by equation 1. The dashed lines indicate the wedge profile as it may actually exist. In the experiments

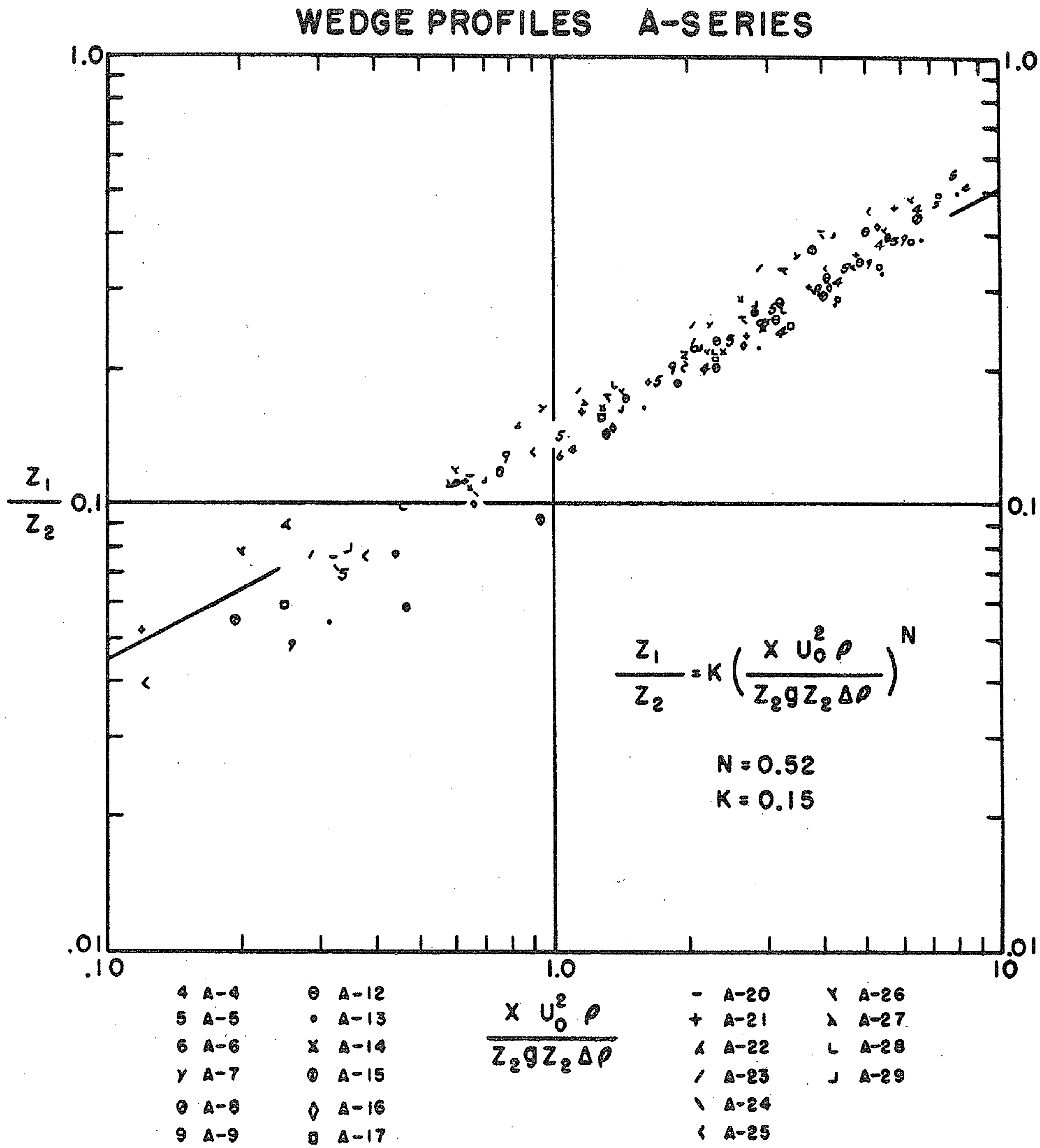

WEDGE PROFILES A-SERIES
$\frac{Z_1}{Z_2}$
$\frac{X\, U_0^2 \rho}{Z_2 g Z_2 \Delta\rho}$
$\frac{Z_1}{Z_2} = K\left(\frac{X\, U_0^2 \rho}{Z_2 g Z_2 \Delta\rho}\right)^N$
N = 0.52
K = 0.15
4 A-4
5 A-5
6 A-6
y A-7
θ A-8
9 A-9
θ A-12
• A-13
X A-14
⊗ A-15
◊ A-16
□ A-17
– A-20
+ A-21
A-22
/ A-23
\ A-24
‹ A-25
Y A-26
λ A-27
L A-28
J A-29

FIG. 9

WEDGE PROFILES B-SERIES

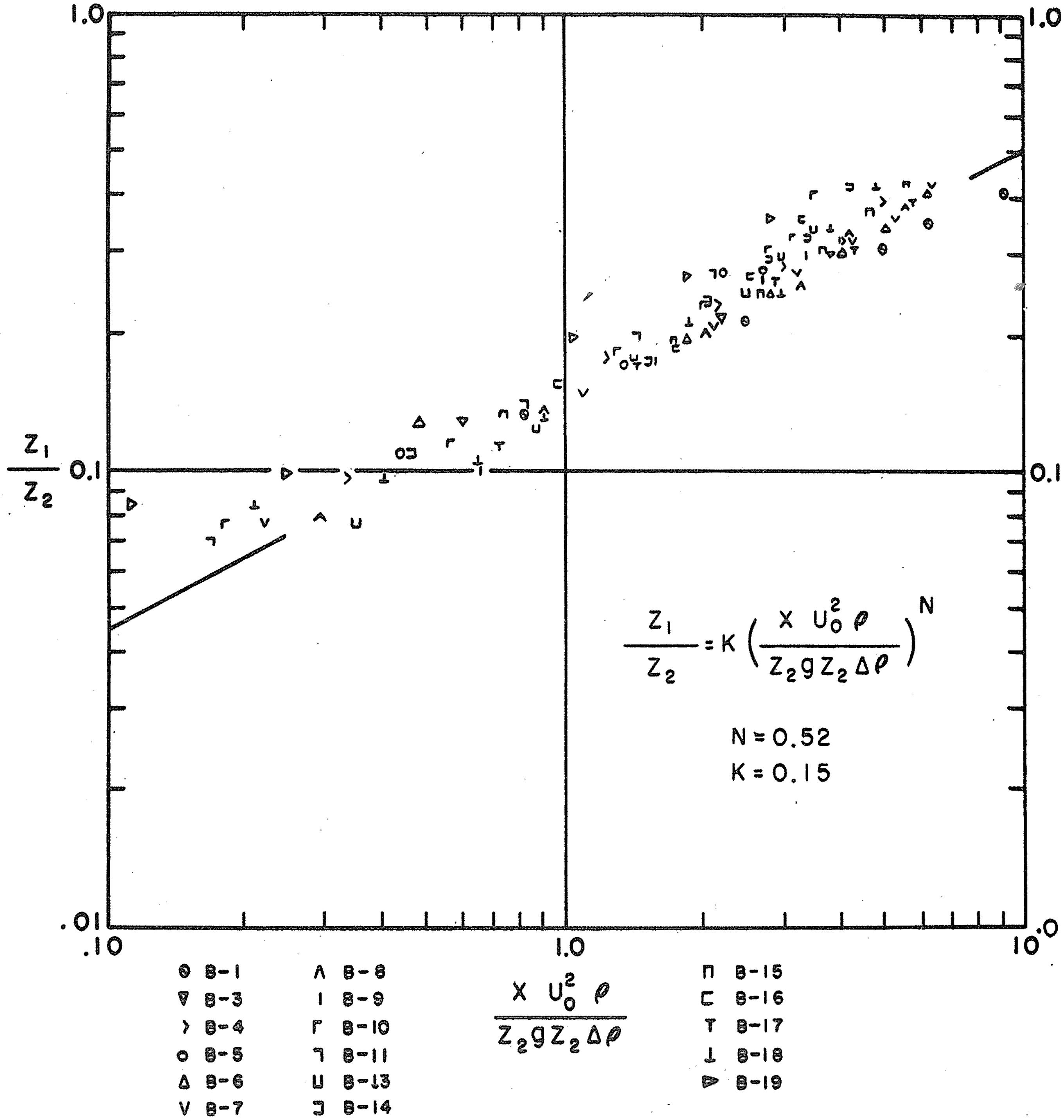


FIG.10

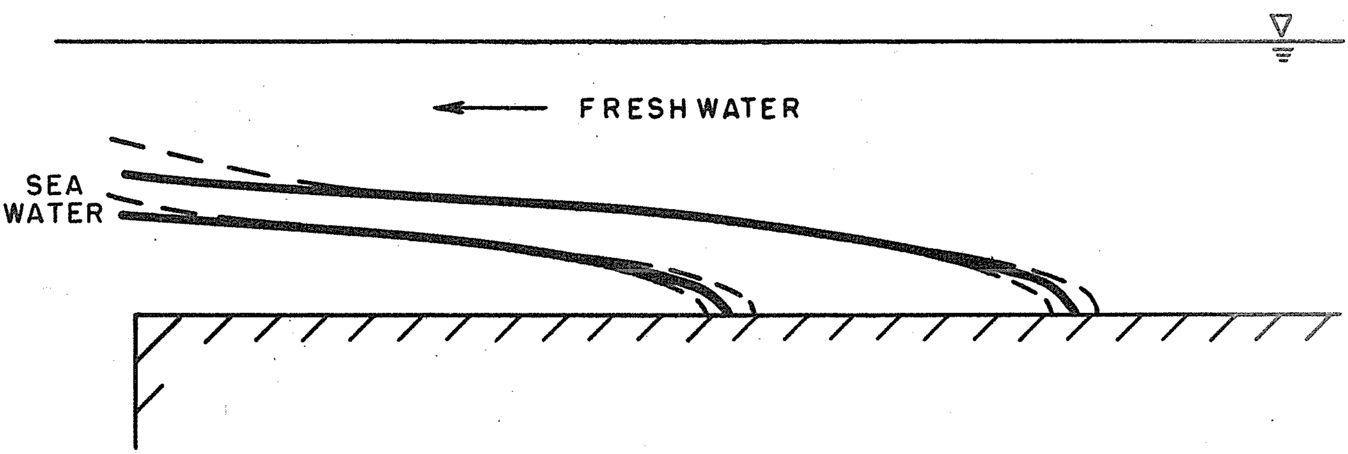


FIG. 11

the leading edge of the wedge was not uniform across the width of the flume. In the center of the flume the wedge was generally 5 cm to 8 cm shorter than at the side walls. The average position of the wedge is that which was recorded. The values of X near the toe of the wedge may have been in error as high as 20% to 30%.

The measurement of Z_1 at the deep end of the wedge is subject to greater error than elsewhere on the wedge because of the diffuse character of the interface. The spread of points is also due to the effect of the interface curving up near the end of the wedge and the wedges terminating at various values of the abscissa.

Another interesting result obtained from the experimental data is that in all cases the corrected Froude number,

$$1/F = \frac{\Delta\rho}{\rho} \frac{g(Z_2 - Z_1)^3}{q^2}$$

approaches unity at the flume to reservoir transition. The abrupt widening of the channel apparently acts as a critical control section to the flow in the upper layer. This result, $F = 1$, may be used to determine the minimum depth of the upper layer and is therefore a limiting boundary condition to equation 1. The computed values of $1/F$ for all the wedges are included with the data in the Appendix.

Application of Experimental Results

The empirical relation, expressed by equation 1, appears to adequately describe the salt wedges as observed in the flume. For the equation to be generally applicable, it should describe salt wedges as found in natural estuaries.

The only estuary containing a salt wedge for which data was obtained, is the Mississippi River. There are three passes through which the Mississippi River water discharges into the Gulf of Mexico. South West Pass is the main channel for navigation and some data on the salt wedge in this Pass was obtained. During an unusual hydrograph when the river discharge decreased markedly over a two weeks period, the salt wedge penetrated well upstream above the head of the passes into the main river channel. Data for the salt wedge under these conditions is also available. Except for the changing hydrograph the salt wedge should be stationary as there is only a negligible tide at the river mouth. The specific data used is that at the end of the two week period. The references suggest that the wedge is well defined, however, no vertical profiles of the salinity and velocity are given. The characteristics of the reverse current within the wedge and the sharpness of the interface are therefore not known. The interface of the salt wedge in South West Pass is defined as the 10 $^{o}/oo$ isohaline and for the wedge above the head of the passes as the 5 $^{o}/oo$ isohaline. In the latter case, $\Delta\rho/\rho$ is given as 0.01, however, it is not known how representative this figure is without more salinity information. In South West Pass $\Delta\rho/\rho$ had to be assumed. Table I and Ia summarize the data and give the length of the wedge as computed by equation 1.

At the end of the jetties of South West Pass, the Froude number approached unity and in this respect the end condition is similar to that as in the laboratory. The end condition is not known for the wedge which has penetrated above the head of the passes. In the main river channel the wedge is not stationary but is slowly moving upstream. In both instances the length of the computed wedge is considerably shorter than that observed in nature. These results were not materially bettered by reasonable adjustments in the data.

Equation 1, however, has yielded very interesting results when applied to estuaries where there is considerable vertical mixing. This application requires several assumptions which should be clearly stated.

The mixing of the fresh and salt water in most estuaries is believed to be due mainly to the tidal currents. These currents are very strong in many estuaries and often predominate over the full depth of water. In these estuaries the velocity and density distribution is entirely different from that of the salt wedge. The point of maximum intrusion of the salt water will move back and forth with the falling and rising tide. It is assumed that at mean tide the intrusion of the salt water is at its mean position and that this corresponds to the length of a stationary salt wedge. It is also assumed that over any vertical cross section the total amount of salt water of ocean density or salinity is the same whether it is well mixed vertically or undiluted as in the salt wedge. To estimate what the depth of undiluted salt water would be at a given station, the total depth of water is multiplied by the ratio of the mean salinity at the station to the salinity of the salt water in the ocean.

Tables II, III and IV summarize the data and results for three estuaries where the fresh and salt water are well mixed. In each case the cross section of the river was reasonably uniform over the length and an average width was estimated. For North Channel, Savannah Harbor, and St. Johns River, Florida, the data on the vertical density distribution as given by the references is quite limited. In the former case it consists of surface and bottom densities over a tidal period for only two stations. In St. Johns River, surface, mid-depth, and bottom densities over a tidal period are given for three stations. The entrances of both rivers into the ocean are confined by jetties. No data is given at the ends of the jetties so in order to estimate the depth of the wedge at these points the corrected Froude number is assumed unity.

The salinity data for the Raritan River, New Jersey, consists of longitudinal cross sections over the full length of the salt intrusion in the river. This data also extends out into Raritan

Bay. A rather complete picture of the salinity structure over the full estuary is therefore known. The entrance of Raritan River into Raritan Bay is more gradual than those above as no jetties have been installed. The depth of the river outside the navigation channel is quite shallow, in the order of one to three feet at MLT.

Table I

Mississippi River

Salt Wedge above Head of Passes

Observed Conditions: (From Reference)

Total fresh water discharge	=	150,000 cu ft/sec
Average width of river	=	2,300 ft
Average depth (estimated)	=	90 ft
$\Delta\rho/\rho$	=	.01
Upstream limit of wedge at 995 mi. below Cairo		
Depth of upper layer at 1040 mi. below Cairo	=	30 ft
Negligible tides		

Results from Empirical Equation:

Length of salt wedge upstream of Station 1040 (computed)	=	16.5 miles
(From Reference, plate 7)	=	45 miles

Reference: Third Progress Report on Model Laws for Density Currents. National Bureau of Standards Hydraulic Laboratory, Washington, D. C. Dec. 6, 1946.

Table Ia

Mississippi River

Salt Wedge in South West Pass

Observed Conditions: (From Reference)		
Total fresh water discharge	=	100,000 cfs
Average width	=	1,500 ft
Average depth	=	45 ft
Negligible tides		
At end of jetties: depth of fresh water layer	=	20 ft
Results of Empirical Equation:		
$\Delta\rho/\rho$ (assumed)	=	.02
Length of salt wedge (computed)	=	1.4 miles
(From Reference)	=	14 miles
At end of jetties the Froude No. 1/F	=	1.15

Reference: Evaluation of Present State of Knowledge of Factors Affecting Tidal Hydraulics and Related Phenomena. Committee on Tidal Hydraulics, Report No. 1, Feb. 1950.

Table II

Raritan River, New Jersey

Observed Conditions: (From Reference)

Total fresh water discharge	=	2,360 cu ft/sec
Average width of channel	=	450 ft
Average mean tidal depth	=	17 ft
Tidal range	=	5.5 ft
$\Delta\rho/\rho$	=	.020
Mean depth of water at South Amboy	=	23 ft
Average salinity at South Amboy	=	21 $^{o}/oo$

Computed Results:

Corrected Froude No. at South Amboy	F =	0.83
Length of salt wedge above South Amboy (computed)	=	7.9 naut. miles
Average length of salt intrusion (From Reference)	=	8.5 naut. miles
Salinity 4 miles upstream (computed)	=	13 $^{o}/oo$
(From Reference)	=	12 $^{o}/oo$

Reference: Hydrographic Considerations Relative to the Location of Sewer Outfall in Raritan Bay, by Ayers, Ketchum, and Redfield. May 1949. Ref. No. 49-13, Woods Hole Oceanographic Institution.

Table III

North Channel, Savannah Harbor

Observed Conditions: (From Reference)

Total fresh water discharge	=	8,200 cu ft/sec
Width of channel	=	500 ft
Depth of channel MLT	=	30 ft
Tidal range	=	7 ft
Mean depth of channel	=	33.5 ft
$\Delta\rho/\rho$	=	.020
Mean density at Station 190	=	1.013
" " " " 143	=	1.003

Computed Results:

At end of jetties, Sta. 205, let corrected Froude No. be unity

Depth of salt water at Sta. 205	=	26 ft
Length of wedge above Sta. 205 (computed)	=	70,500 ft
" " " " " 190 (")	=	51,000 ft
" " " " " 143 (")	=	3,000 ft
Computed distance between Sta. 205-190	=	19,500 ft
true distance	=	15,000 ft
Computed distance between Sta. 205-143	=	67,500 ft
true distance	=	62,000 ft
Computed distance between Sta. 190-143	=	48,000 ft
true distance	=	47,000 ft

Reference: Plans for Improvement of Navigation Conditions and Elimination of Shoaling in Savannah Harbor. Technical Memo No. 2-268, Vol. I and II, Waterways Experiment Station, Vicksburg, Miss. March 1949.

Table IV

<u>St. Johns River, Florida</u>

Observed Conditions: (From Reference)

Total fresh water discharge	=	17,000 cu ft/sec
Average width of river	=	900 ft
Average depth of water (MLT)	=	30 ft
Tidal range	=	4 ft
Mean depth of water	=	32 ft
$\Delta\rho/\rho$	=	.025

Computed Results:

Corrected Froude number equals unity at end of jetty		
Depth of salt water at end of jetty	=	24.4 ft
Length of salt wedge (computed)	=	8.9 naut. miles
Length of salt wedge (From Reference)	=	9-10 naut. miles
Average salinity at Fulton (computed)	=	9.7 $^{o}/oo$
Average salinity at Fulton (From Reference)	$\approx$	5 $^{o}/oo$

(In vicinity of Fulton 3 large streams enter St. Johns River which would in effect reduce the salinity.)

Reference: Plans for the Improvement of the St. Johns River Jacksonville to the Atlantic Ocean, Technical Memo No. 2-244, Waterways Experiment Station, Vicksburg, Miss. Dec. 1947.

Discussion and Results

The results obtained from equation 1 for the Mississippi River salt wedge are not in good agreement with the data supplied by the references. For South West Pass of the Mississippi River the end condition and shape of the channel are satisfactory. The poor agreement may therefore be due to an improper scaling of the friction effects. The shear stresses are greatest and believed most significant at the channel bottom and at the interface. The coefficient k includes these effects and has been assumed constant. As the computed length of the salt wedge is shorter than that which actually occurs, these shear stresses appear to be reduced. Also, within the prototype wedge the fluid motions are turbulent whereas in the laboratory they are predominantly laminar.

Before any more definite conclusions can be stated more data on salt wedges in natural estuaries should be secured and equation 1 checked further.

In the other three estuaries investigated the fresh and salt waters were well mixed. It is rather surprising that the mean horizontal distribution of salinity, determined by equation 1, compares favorably to the actual distribution. These results cannot be explained at the present time. The assumptions concerning the salinity distribution are not necessarily justified as the velocities in the two systems are dissimilar.

A theoretical treatment of the stationary salt wedge is presently in progress and will be submitted at a later date.

APPENDIX

WEDGE PROFILE DATA

A Series

$\rho = 1.000$ $\rho_1 = 1.025$ $\frac{\Delta\rho}{\rho} = .025$

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X\,U_o^2\,\rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2-Z_1)^3}{\rho\quad q^2}$
TEST A-3		Z_2 = 3.40 cm.		U_o = 4.20 cm/sec	
30.5	.17	.050	9.0	1.91	4.07
61.0	.70	.206	17.9	3.79	2.36
91.5	1.18	.348	26.9	5.70	1.32
TEST A-4		Z_2 = 3.91 cm.		U_o = 3.65 cm/sec	
30.5	.52	.133	7.8	1.08	4.71
61.0	.79	.202	15.6	2.16	3.65
91.5	.95	.243	23.4	3.25	3.15
122.	1.22	.312	31.2	4.34	2.33
152.	1.50	.384	38.9	5.40	1.71
182.	1.78	.455	46.5	6.45	1.16
TEST A-5		Z_2 = 4.38 cm.		U_o = 3.26 cm/sec	
15.0	.30	.069	3.4	.333	8.31
46.0	.62	.142	10.5	1.03	6.45
76.0	.82	.187	17.4	1.71	5.37
107.	1.02	.233	24.4	2.46	4.61
137.	1.20	.274	31.4	3.08	3.92
168.	1.35	.308	38.4	3.76	3.38
198.	1.47	.336	45.3	4.44	2.94
229.	1.64	.374	52.4	5.14	2.50
259.	1.70	.388	59.2	5.80	2.34
290.	1.87	.426	66.2	6.49	1.89
320.	2.05	.469	73.1	7.16	1.59
351.	2.38	.543	80.2	7.86	.96
TEST A-6		Z_2 = 3.94 cm.		U_o = 5.08 cm/sec	
15.0	.50	.127	3.8	1.02	2.52
30.0	.88	.223	7.6	2.04	1.79
TEST A-7		Z_2 = 4.42 cm.		U_o = 4.53 cm/sec	
22.0	.72	.163	5.0	.944	3.12
52.0	1.10	.250	11.8	2.22	2.19
83.0	1.58	.358	18.8	3.54	1.43

$\rho = 1.000 \qquad \rho_1 = 1.025 \qquad \frac{\Delta\rho}{\rho} = .025$

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X U_o^2 \rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2 - Z_1)^3}{\rho \quad q^2}$
TEST A-8		Z_2 = 4.93 cm.		U_o = 4.06 cm/sec	
7.0	.27	.0548	1.42	.194	6.30
22.0	.55	.112	4.47	.611	5.19
52.0	.85	.173	10.5	1.44	4.15
83.0	1.12	.227	16.8	2.30	3.42
113.	1.34	.272	22.9	3.13	2.88
144.	1.45	.294	29.2	4.00	2.58
174.	1.72	.350	35.4	4.74	2.05
205.	1.90	.386	41.6	5.69	1.71
235.	2.18	.442	47.7	6.53	1.29
TEST A-9		Z_2 = 5.54 cm.		U_o = 3.62 cm/sec	
15.0	.27	.0487	2.71	.262	8.94
45.0	.72	.130	8.12	.785	6.85
106.	1.12	.202	19.1	1.84	5.34
167.	1.42	.256	30.2	2.92	4.28
228.	1.67	.302	41.2	3.98	3.55
289.	1.90	.343	52.3	5.05	2.73
350.	2.19	.395	63.3	6.1	2.04
411.	2.42	.437	74.1	7.15	1.87
472.	2.78	.503	85.1	8.21	1.29
TEST A-10		Z_2 = 4.55 cm.		U_o = 5.65 cm/sec	
7.0	.45	.0989	1.54	.441	2.56
23.0	.93	.204	5.05	1.45	1.76
TEST A-11		Z_2 = 4.95 cm.		U_o = 5.20 cm/sec	
7.0	.42	.0848	1.41	.314	3.44
37.0	1.00	.202	7.47	1.67	2.26
68.0	1.58	.319	13.7	3.05	1.41
TEST A-12		Z_2 = 5.50 cm.		U_o = 4.68 cm/sec	
15.0	.42	.0765	2.73	.444	4.91
45.0	.79	.144	8.19	1.33	3.90
76.0	1.10	.200	13.8	2.24	3.26
106.	1.42	.258	19.3	3.14	2.52
137.	1.75	.319	24.9	4.05	1.97
168.	2.23	.406	30.6	4.97	1.29

$\rho = 1.000 \qquad \rho_1 = 1.025 \qquad \frac{\Delta\rho}{\rho} = .025$

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X\,U_o^2\,\rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2-Z_1)^3}{\rho \quad q^2}$
TEST A-13		Z_2 = 5.97 cm.		U_o = 4.30 cm/sec	
15.0	.32	.0536	2.51	.316	6.71
76.0	.97	.162	12.7	1.60	4.64
137.	1.32	.221	22.9	2.89	3.70
198.	1.65	.276	33.2	4.27	3.00
259.	1.94	.325	43.4	5.46	2.41
320.	2.32	.388	53.5	6.74	1.81
381.	2.93	.490	63.8	8.04	1.04
TEST A-14		Z_2 = 5.57 cm		U_o = 5.66 cm/sec	
15.0	.62	.111	2.70	.635	3.02
30.0	.90	.161	5.39	1.27	2.56
61.0	1.58	.284	10.9	2.56	1.58
TEST A-15		Z_2 = 6.02 cm		U_o = 5.25 cm/sec	
15.0	.35	.0582	2.50	.467	4.51
30.0	.55	.0915	4.99	.933	4.06
61.0	1.12	.186	10.1	1.89	2.92
91.0	1.65	.274	15.1	2.82	2.08
122.	2.23	.370	20.3	3.80	1.35
TEST A-16		Z_2 = 6.56 cm.		U_o = 4.81 cm/sec	
30.0	.65	.099	4.57	.659	5.10
61.0	.95	.145	9.30	1.34	4.35
122.	1.49	.227	18.6	2.68	3.24
183.	2.07	.316	27.9	4.02	2.24
244.	2.73	.416	37.2	5.35	1.39
TEST A-17		Z_2 = 7.04 cm.		U_o = 4.49 cm/sec	
15.0	.42	.0595	2.13	.249	7.16
45.0	.82	.117	6.4	.749	5.94
76.0	1.12	.159	10.8	1.26	5.15
137.	1.47	.209	19.5	2.28	4.28
198.	1.72	.244	28.2	3.30	3.73
259.	2.00	.284	36.8	4.30	3.16
320.	2.34	.332	45.5	5.33	2.57
381.	2.67	.380	54.1	6.34	2.08
442.	3.38	.480	62.8	7.35	1.22

$\rho = 1.000$ $\rho_1 = 1.025$ $\frac{\Delta\rho}{\rho} = .025$

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X U_o^2 \rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2-Z_1)^3}{\rho \quad q^2}$
TEST A-18		$Z_2 = 6.15$ cm.		$U_o = 6.05$ cm/sec	
15.0	.45	.0732	2.46	.597	3.27
30.0	.85	.138	4.87	1.21	2.64
61.0	1.53	.248	9.91	2.47	1.71
TEST A-19		$Z_2 = 6.65$ cm.		$U_o = 5.60$ cm/sec	
7.0	.45	.0676	1.05	.202	4.23
37.0	1.12	.169	5.56	1.07	3.02
68.0	1.60	.240	10.2	1.96	2.28
98.0	2.28	.343	14.7	2.83	1.49
TEST A-20		$Z_2 = 7.15$ cm.		$U_o = 5.20$ cm/sec	
15.0	.54	.0755	2.10	.324	5.14
30.0	.82	.115	4.20	.649	4.49
61.0	1.24	.174	8.53	1.32	3.67
91.0	1.55	.217	12.7	1.96	3.12
122.	1.87	.262	17.2	2.66	2.60
152.	2.40	.336	21.3	3.29	1.89
183.	2.88	.404	25.6	3.95	1.38
TEST A-21		$Z_2 = 7.60$ cm.		$U_o = 4.90$ cm/sec	
7.0	.40	.0526	.922	.119	6.58
37.0	.87	.114	4.87	.629	5.41
68.0	1.22	.161	8.95	1.15	4.61
98.0	1.42	.187	12.9	1.66	4.17
159.	1.80	.237	20.9	2.70	3.45
220.	2.29	.302	29.0	3.74	2.66
281.	2.77	.364	37.1	4.79	2.00
342.	3.48	.458	45.0	5.81	1.25
TEST A-22		$Z_2 = 6.67$ cm.		$U_o = 6.43$ cm/sec	
7.0	.60	.0900	1.05	.252	2.98
22.0	1.00	.150	3.30	.834	2.45
52.0	1.78	.267	7.80	1.97	1.56

$\rho = 1.000$ $\rho_1 = 1.025$ $\frac{\Delta\rho}{\rho} = .025$

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X\,U_o^2\,\rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2-Z_1)^3}{\rho \quad q^2}$
TEST A-23		$Z_2 = 7.17$ cm		$U_o = 5.98$ cm/sec	
10.0	.55	.0765	1.40	.285	3.26
40.0	1.27	.177	5.57	1.13	2.74
71.0	1.80	.251	9.90	2.04	2.06
101.	2.38	.332	14.1	2.87	1.46
TEST A-24		$Z_2 = 7.68$ cm.		$U_o = 5.60$ cm/sec	
15.0	.54	.0704	1.95	.325	4.83
30.0	.82	.107	3.90	.650	4.30
61.0	1.34	.175	7.95	1.32	3.38
91.0	1.60	.208	11.9	1.99	2.98
122.	1.97	.256	15.9	2.66	2.48
152.	2.55	.332	19.8	3.30	1.69
183.	3.08	.401	23.8	3.97	1.30
TEST A-25		$Z_2 = 8.15$ cm.		$U_o = 5.27$ cm/sec	
7.0	.32	.0393	.86	.12	6.39
22.0	.62	.076	2.70	.376	5.71
52.0	1.07	.131	6.40	.893	4.73
113.	1.65	.202	13.9	1.94	3.66
174.	2.04	.251	21.4	2.98	3.05
235.	2.72	.334	28.8	4.02	2.13
296.	3.68	.452	36.4	5.07	1.19
TEST A-26		$Z_2 = 8.66$ cm.		$U_o = 4.95$ cm/sec	
15.0	.67	.0775	1.73	.200	6.79
45.0	1.02	.118	5.20	.600	5.82
106.	1.52	.176	12.2	1.41	4.84
167.	1.87	.216	19.3	2.22	4.15
228.	2.22	.256	26.4	3.04	3.54
289.	2.60	.300	33.4	3.86	2.98
350.	2.94	.340	40.5	4.67	2.48
411.	3.52	.407	47.5	5.49	1.81
472.	4.13	.476	54.5	6.29	1.24

$\rho = 1.000$ $\rho_1 = 1.025$ $\frac{\Delta\rho}{\rho} = .025$

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X U_o^2 \rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2 - Z_1)^3}{\rho \quad q^2}$
TEST A-27		Z_2 = 7.20 cm.		U_o = 7.05 cm/sec	
15.0	.80	.111	2.08	.585	2.49
30.0	1.20	.167	4.17	1.18	2.04
61.0	1.93	.268	8.50	2.40	1.39
TEST A-28		Z_2 = 7.70 cm.		U_o = 6.60 cm/sec.	
15.0	.75	.0975	1.95	.451	3.22
45.0	1.42	.184	5.85	1.35	2.34
76.0	2.05	.266	9.85	2.28	1.72
107.	2.58	.335	13.9	3.22	1.28
TEST A-29		Z_2 = 8.20 cm		U_o = 6.20 cm/sec	
15.0	.64	.078	1.83	.350	4.11
30.0	.92	.112	3.66	.700	3.79
61.0	1.34	.163	7.45	1.43	3.06
91.0	1.80	.22	11.1	2.12	2.49
122.	2.27	.277	14.9	2.85	1.99
183.	3.23	.394	22.3	4.26	1.17

WEDGE PROFILE DATA

B Series

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\dfrac{X\,U_o^2\,\rho}{Z_2 g Z_2 \Delta\rho}$	$\dfrac{\Delta\rho g (Z_2 - Z_1)^3}{\rho \quad q^2}$
TEST B-1		$Z_2 = 5.42$ cm.		$U_o = 3.54$ cm/sec	
	$\rho = 1.0066$		$\rho_1 = 1.0235$	$\frac{\Delta\rho}{\rho} = .0170$	
30.0	.74	.136	5.54	0.81	4.66
91.0	1.15	.212	16.8	2.46	3.62
183.	1.68	.310	33.8	4.95	2.38
229.	1.87	.345	42.2	6.18	2.03
335.	2.25	.415	61.8	9.05	1.45
396.	2.38	.439	73.0	10.7	1.32
457.	2.67	.492	84.2	12.3	.934
TEST B-2		$Z_2 = 5.40$ cm.		$U_o = 2.63$ cm/sec	
	$\rho = 1.0114$		$\rho_1 = 1.0235$	$\frac{\Delta\rho}{\rho} = .0119$	
20.0	.83	.154	3.70	.407	5.55
81.0	1.26	.233	15.0	1.65	4.10
142.	1.56	.289	26.3	2.89	3.30
203.	2.06	.382	37.6	4.14	2.17
TEST B-3		$Z_2 = 6.35$ cm.		$U_o = 2.22$ cm/sec	
	$\rho = 1.0186$		$\rho_1 = 1.0235$	$\frac{\Delta\rho}{\rho} = .00481$	
23.0	.82	.129	3.62	.596	4.00
84.0	1.39	.219	13.2	2.18	2.90
145.	1.90	.300	22.8	3.76	2.09
TEST B-4		$Z_2 = 9.44$ cm		$U_o = 2.09$ cm/sec	
	$\rho = 1.0200$		$\rho_1 = 1.0235$	$\frac{\Delta\rho}{\rho} = .00343$	
23.0	.92	.097	2.44	.337	5.35
84.0	1.67	.177	8.90	1.23	4.05
145.	2.20	.233	15.4	2.12	3.28
206.	2.69	.285	21.8	3.00	2.76
277.	3.03	.321	29.3	4.05	2.28
338.	3.70	.392	35.8	4.95	1.63

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X\,U_o^2\,\rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2-Z_1)^3}{\rho \quad q^2}$
TEST B-5					

Z_2 = 7.40 cm. U_o = 3.07 cm/sec

ρ = 1.0174 ρ_1 = 1.0235 $\frac{\Delta\rho}{\rho}$ = .00599

15.0	.82	.111	2.03	.439	3.24
46.0	1.28	.173	6.21	1.34	2.60
76.0	2.01	.272	10.2	2.2	1.77
94.0	2.04	.276	12.7	2.74	1.75

TEST B-6 Z_2 = 8.40 cm. U_o = 3.04 cm/sec

ρ = 1.0162 ρ_1 = 1.0235 $\frac{\Delta\rho}{\rho}$ = .00718

25.4	.94	.127	3.02	.472	4.52
86.0	1.63	.194	11.6	1.82	3.36
147.	2.02	.240	17.5	2.74	2.82
208.	2.55	.304	24.8	3.89	2.16
269.	2.87	.342	32.0	5.01	1.83
330.	3.41	.406	39.3	6.15	1.34

TEST B-7 Z_2 = 8.44 cm. U_o = 3.37 cm/sec

ρ = 1.0138 ρ_1 = 1.0235 $\frac{\Delta\rho}{\rho}$ = .00956

12.7	.65	.077	1.51	.218	5.46
63.5	1.25	.148	7.50	1.08	4.29
124.	1.78	.211	14.7	2.12	3.42
185.	2.28	.270	21.9	3.17	2.71
246.	2.68	.318	29.2	4.22	2.22
307.	2.97	.352	36.4	5.26	1.90
368.	3.48	.412	43.6	6.3	1.42

TEST B-8 Z_2 = 8.45 cm. U_o = 4.05 cm/sec

ρ = 1.0112 ρ_1 = 1.0235 $\frac{\Delta\rho}{\rho}$ = .0122

15.0	.68	.0805	1.78	.289	4.80
46.0	1.11	.131	5.45	.885	4.04
107.	1.75	.207	12.7	2.06	3.16
168.	2.16	.256	19.9	3.23	2.54
229.	2.77	.328	27.2	4.41	1.87
291.	3.25	.385	34.5	5.60	1.56

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X U_o^2 \rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2-Z_1)^3}{\rho \quad q^2}$
TEST B-9		Z_2 = 8.51 cm		U_o = 4.66 cm/sec	
	ρ = 1.0094		ρ_1 = 1.0235	$\frac{\Delta\rho}{\rho}$ = .0140	
30.0	.95	.101	3.52	.656	3.86
61.0	1.50	.176	7.16	1.54	3.02
122.	2.24	.264	14.4	2.68	2.14
152.	2.53	.298	17.9	3.34	1.86
183.	2.70	.318	21.5	4.01	1.72
TEST B-10		Z_2 = 13.98 cm.		U_o = 2.83 cm/sec	
	ρ = 1.0186		ρ_1 = 1.0222	$\frac{\Delta\rho}{\rho}$ = .00352	
15.0	1.07	.0766	1.07	1.79	4.74
46.0	1.62	.116	3.30	.55	9.15
107.	2.55	.183	7.66	1.28	3.29
168.	3.26	.234	12.0	2.00	2.72
229.	4.23	.303	16.4	2.74	2.04
259.	4.54	.325	18.6	3.11	1.85
289.	5.66	.405	20.7	3.46	1.26
TEST B-11		Z_2 = 14.07 cm		U_o = 2.61 cm/sec	
	ρ = 1.0218		ρ_1 = 1.0235	$\frac{\Delta\rho}{\rho}$ = .00166	
8.0	.99	.0703	.568	.169	2.68
38.0	2.02	.143	2.70	.805	2.10
68.0	2.80	.199	4.83	1.43	1.72
99.0	3.83	.272	7.03	2.09	1.30
TEST B-13		Z_2 = 11.07 cm.		U_o = 2.66 cm/sec	
	ρ = 1.0200		ρ_1 = 1.0235	$\frac{\Delta\rho}{\rho}$ = .00343	
20.0	.84	.0758	1.81	.345	4.20
50.0	1.35	.122	4.52	.86	3.58
81.0	1.90	.172	7.31	1.40	3.01
142.	2.68	.242	12.8	2.44	2.29
172.	3.22	.291	15.5	2.95	1.88
203.	3.71	.335	18.3	3.48	1.55

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X\, U_o^2\, \rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2 - Z_1)^3}{\rho \quad q^2}$
TEST B-14	Z_2 = 12.00 cm.		U_o = 2.48 cm/sec		
	ρ = 1.0200		ρ_1 = 1.0235	$\frac{\Delta\rho}{\rho}$ = .00343	
36.0	1.31	.109	3.00	.458	4.61
97.0	2.09	.174	8.08	1.53	3.70
158.	2.78	.232	13.2	2.02	2.96
219.	3.53	.294	18.2	2.78	2.30
270.	3.87	.322	22.5	3.43	1.66
331.	5.08	.423	27.6	4.21	1.25
TEST B-15	Z_2 = 8.80 cm.		U_o = 2.57 cm/sec		
	ρ = 1.0180		ρ_1 = 1.0236	$\frac{\Delta\rho}{\rho}$ = .0055	
46.0	1.17	.133	5.23	.731	4.65
107.	1.69	.192	12.2	1.71	3.80
168.	2.13	.242	19.1	2.68	3.12
229.	2.70	.307	26.0	3.64	2.39
290.	3.28	.373	33.0	4.62	1.78
351.	3.77	.428	39.9	5.58	1.34
TEST B-16	Z_2 =7.81 cm		U_o = 2.57 cm/sec		
	ρ = 1.0180		ρ_1 = 1.0236	$\frac{\Delta\rho}{\rho}$ = .0055	
38.0	1.21	.155	4.86	.96	3.04
68.0	1.50	.192	8.70	1.72	2.66
99.0	2.09	.268	12.7	2.52	1.99
129.	2.78	.356	16.5	3.28	1.33
TEST B-17	Z_2 = 6.86 cm		U_o = 2.47 cm/sec		
	ρ = 1.0179		ρ_1 = 1.0236	$\frac{\Delta\rho}{\rho}$ = .0056	
30.0	.78	.114	4.37	.707	4.32
61.0	1.17	.171	8.90	1.44	3.53
122.	1.80	.262	17.8	2.88	2.50
183.	2.11	.308	26.7	4.32	2.05
244.	2.68	.390	35.6	5.76	1.40

X (cm)	Z_1 (cm)	Z_1/Z_2	X/Z_2	$\frac{X U_o^2 \rho}{Z_2 g Z_2 \Delta\rho}$	$\frac{\Delta\rho g (Z_2-Z_1)^3}{\rho \quad q^2}$
TEST B-18	Z_2 = 10.80 cm		U_o = 2.47 cm/sec		
	ρ = 1.0180		ρ_1 = 1.0235	$\frac{\Delta\rho}{\rho}$ = .0054	
13.0	.90	.0833	1.20	.207	4.50
25.0	1.03	.0952	2.31	.400	4.31
40.0	1.12	.104	3.70	.64	4.16
56.0	1.39	.129	5.18	.895	3.84
117.	2.26	.209	10.8	1.86	2.84
178.	2.62	.242	16.5	2.85	2.51
239.	3.66	.338	22.1	3.82	1.66
300.	4.53	.419	27.8	4.80	1.14
TEST B-19	Z_2 = 28.00 cm		U_o = 1.82 cm/sec		
	ρ = 1.0233		ρ_1 = 1.0240	$\frac{\Delta\rho}{\rho}$ = .000684	
17.8	2.38	.085	.635	.112	4.35
38.1	2.78	.099	1.36	.24	4.15
162.6	5.44	.194	5.8	1.02	2.94
292.	7.48	.267	10.4	1.83	2.22
437.	10.1	.36	15.6	2.75	1.24

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